Eur. Phys. J. C 56, 147–151 (2008)
DOI 10.1140/epjc/s10052-008-0635-1

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

Scaling anomaly in string defect background

P.R. Giri[a]

Theory Division, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Calcutta 700064, India

Received: 25 December 2007 / Revised version: 23 April 2008 /
Published online: 7 June 2008 –

Abstract. We show that the classical scale symmetry of a particle moving in string defect background is broken due to inequivalent quantization of the classical system, which leads to scaling anomaly. The consequence of this anomaly is the formation of single bound state in the coupling constant interval $\gamma \in (-1, 1)$. The inequivalent quantization is characterized by a 1-parameter family of self-adjoint extension parameter ω. It has been conjectured that the formation of loosely bound state in string defect background may lead to the so called anomalous scattering cross section for the particles, which has been experimentally observed in molecular physics. A plausible laboratory test for the anomalous scattering could be devised in condensed matter system.

PACS. 03.65.-w; 03.65.Db; 98.80.Cq; 11.27.+d

Symmetry and corresponding symmetry breaking [1] is an extremely important issue in physics because of their consequences in different physical processes. Usually in physics we consider three different sorts of symmetry breaking, i.e., spontaneous, explicit and anomalous symmetry breaking. However, anomalous symmetry breaking [2, 3] occurs when some kind of classical invariance of a system is violated upon quantization. In quantum field theory [3–7] and string theory [8] anomaly is a hugely studied issue. From the theoretical point of view anomaly is being investigated in various fields starting from molecular physics [9, 10] to black hole [11]. In quantum mechanics an operator becomes anomalous when it does not keep the domain of the Hamiltonian invariant. There is another interesting example of anomaly, which occurs in molecular physics [9, 10] in quantum mechanical context. For example, interaction of an electron in the field of a polar molecule is a simple example of anomaly, where the classical scaling symmetry of the system is broken once it goes inequivalent quantization [12]. The obvious consequence of this scaling anomaly in molecular physics is the occurrence of bound state and the dependence of momentum in the phase shift of scattering cross section.

The problem of quantum anomaly which we consider here may occur in the background of a string defect, when a particle is moving in it. This problem has received lots of interest due to its analogy [13–15] with Aharanov–Bohm effect [16]. In relativistic theory it has been shown [17, 18] that the Dirac equation in cosmic string background needs nontrivial boundary condition to be imposed on the spinor wave-function at the origin. See other examples also [19–21]. In language of mathematics the construction of nontrivial boundary condition is usually called self-adjoint extensions [22–24]. The extensions can be characterized by independent parameters and different value of parameters lead to inequivalent theories. It has been observed [25] that in cosmic string scenario the fermionic charge can be non-integral multiple of Higgs charge. Since the flux is quantized with respect to the Higgs charge, it will lead to nontrivial Aharanov–Bohm scattering of fermion. This result has Phenomenological importance because, the cross section is much larger than the one coming from gravitational scattering. In this letter we will discuss about a possible enhancement of scattering cross section due to the temporary formation of loosely bound state in background space of string defect.

In non-relativistic theory [26, 27], the consideration of inequivalent quantization is also inevitable in order to get bound state for the particle moving in string defect background. In [28] gravitational scattering by particles of a spinning source in two dimension has been studied. There, it has been shown that the energy eigenvalue and corresponding eigenfunction of a particle in the field of a massless spinning source is equivalent to that in a background Aharanov–Bohm gauge field of an infinitely thin flux tube. This topological effect also appears in elastic solids [29–32]. The anomaly which we will discuss here has the consequences, which is known for quite a some time. But, the issue of anomaly in string defect background has remained unnoticed as far as we know. Most importantly, despite the similarity with electron polar molecule system, there is no discussion in literature about the possible anomaly in scattering cross section, which is usually seen in molecular physics [12].

This letter has been organized in the following way: First, we study the scaling symmetry of the classical sys-

[a] e-mail: pulakranjan.giri@saha.ac.in

tem, which undergoes anomalous symmetry breaking upon quantization; Second, we made an inequivalent quantization of the system, which is responsible for anomaly and discuss its consequences; Third, we draw an analogy with molecular physics and conjecture that in string defect background there may have anomalous scattering cross section due to the loosely bound state.

First, what is scaling symmetry and why it is important in string defect background? Scale transformation can be defined by the transformation $t \to \beta^2 t$, $\mathbf{r} \to \beta \mathbf{r}$ [33], where β is the scaling factor. Note that the transformation of space co-ordinates and time in non-relativistic theory is treated differently, the details of which can be found in [33]. In classical physics when the action is invariant under this transformation, then the corresponding system has scale symmetry. Since in non-relativistic quantum theory, string defect induces a $1/r^2$ potential to the the particle moving in its background, the relevant classical symmetry would be the scale symmetry. To be more specific, the Hamiltonian for the system $H = \frac{\mathbf{p}^2}{2M}$, scales as $H \to \frac{1}{\beta^2} H$. The scale invariance of this Hamiltonian means, if ψ is an eigenstate of the Hamiltonian H with eigenvalue E, i.e., $H\psi = E\psi$, then $\psi_\beta = \psi(\beta \mathbf{r})$ will also be an eigenstate of the same Hamiltonian with energy E/β^2. This essentially means that the system with scale symmetry does not have any lower bound in energy; that means it cannot have any bound state. Scale symmetry associated with the generator D belongs to the conformal symmetry $\mathrm{SO}(2,1)$ formed by three generators: the Hamiltonian H, the dilatation generator $D = tH - \frac{1}{4}(\mathbf{r}.\mathbf{p} + \mathbf{p}.\mathbf{r})$ and the conformal generator $K = Ht^2 - \frac{1}{2}(\mathbf{r}.\mathbf{p} + \mathbf{p}.\mathbf{r}) + \frac{1}{2} M \mathbf{r}^2$. The commutator algebra for $\mathrm{SO}(2,1)$ is: $[D,H] = -\mathrm{i}\hbar H$, $[D,K] = \mathrm{i}\hbar K$, $[H,K] = 2\mathrm{i}\hbar D$ [34]. In string defect case this scale symmetry has not been noticed so far.

Second, we consider a non-relativistic particle of mass M, moving in the background of string defect. The background is described by the space-time metric in cylindrical coordinate (r, ϕ, z) as [29–32]

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}z^2 - \mathrm{d}r^2 - \alpha^2 r^2 \,\mathrm{d}\phi^2 \,, \tag{1}$$

where α characterizes the string. The constant α introduces an angular deficit of $2\pi(1-\alpha)$ in the Minkowski space-time and is responsible for inducing scale invariant $1/r^2$ potential in non-relativistic quantum system. Due to cylindrical symmetry of the space, we can easily see that the motion of the particle in the z direction is basically a free particle motion, described by the wave-function $\mathrm{e}^{\mathrm{i}kz}$. k is wave-vector of the particle along the z direction. Since we are considering an infinite string defect along the z direction, it is enough to discuss the motion of the particle on the plane perpendicular to the z direction. The motion of the particle on the plane perpendicular to the z axis is described by the time independent Schrödinger equation

$$-\frac{\hbar^2}{2M}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{\alpha^2 r^2}\frac{\partial^2}{\partial \phi^2}\right]\Psi = E\Psi \,. \tag{2}$$

Exploiting the periodicity condition [35]

$$\Psi(\phi + 2\pi) = \mathrm{e}^{2\pi\lambda \mathrm{i}}\Psi(\phi)\,, \tag{3}$$

where $\lambda \in [0,1)$, the wave-function can be separated as $\Psi(r,\phi) = R(r)\,\mathrm{e}^{\mathrm{i}(m+\lambda)\phi}$ and (2) gives the radial equation

$$H_{\mathrm{r}} R(r) \equiv -\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} + \frac{\gamma^2}{r^2}\right] R(r) = \mathcal{E} R(r)\,, \tag{4}$$

where H_{r} is the radial Hamiltonian, with eigenvalue $\mathcal{E} = \frac{2ME}{\hbar^2}$, $\gamma = \frac{m+\lambda}{\alpha}$ and $m = 0, \pm1, \pm2, \cdots$. We will now discuss the solution of the Hamiltonian H_{r}. To discuss that we need to know some general property of an operator, let say $\mathcal{O}$. For the moment let us restrict ourself to the case of unbounded operator, because the Hamiltonian we are discussing is unbounded from below. Now, it is known that [22, 23] for an unbounded operator $\mathcal{O}$, we first need to define the domain $D(\mathcal{O})$. This allows us to construct the adjoint operator $\mathcal{O}^*$ and the corresponding domain $D(\mathcal{O}^*)$. By definition, $\mathcal{O}$ is self-adjoint if and only if $D(\mathcal{O}) = D(\mathcal{O}^*)$ [22, 23]. It is also possible to get a more technical and mathematical analysis for the criteria of self-adjointness of the operator $\mathcal{O}$ in terms of deficiency indices $n_\pm$. Let $\mathcal{K}_\pm = Ker(i \pm \mathcal{O}^*)$, where $Ker(X)$ is the kernel of the operator. The deficiency indices $n_\pm$ are the dimension of the kernel $\mathcal{K}_\pm$. If $n_\pm = 0$, then the operator $\mathcal{O}$ is essentially self-adjoint. If $n_+ = n_- = n \neq 0$, then $\mathcal{O}$ is not self-adjoint but admits self-adjoint extensions and these self-adjoint extensions are identified by the elements of an $U(n)$ group. The operator $\mathcal{O}$ on the other hand cannot be made self-adjoint if $n_+ \neq n_-$.

Let us now come back to the discussion of our operator of interest, which is H_{r}. The Hamiltonian H_{r} is defined on the domain $\mathcal{L}^2[R^+, rdr]$. Classically, this system is scale invariant, because the coupling constant γ of the inverse square potential is a dimensionless constant. However, quantum mechanical analysis of this operator is much more subtle. The Hamiltonian H_{r} is essentially self-adjoint only for $\gamma^2 \geq 1$ in the domain

$$\mathcal{D}_0 = \{\psi \in \mathcal{L}^2(rdr), \psi(0) = \psi'(0) = 0\}\,, \tag{5}$$

of the Hamiltonian H_{r}. For $\gamma \in (-1,1)$, the Hamiltonian is not essentially self-adjoint and therefore cannot play a role for the Hamiltonian (it is therefore called formal Hamiltonian [22, 23]) and so has to be extended to another operator. Note that only $m = 0, \lambda \in [0,1)$ and $m = -1, \lambda \in (0,1)$ belong to the interval $\gamma \in (-1,1)$. For this case the deficiency indices are $(1,1)$, and so the self-adjoint extensions are labeled by a $\mathrm{U}(1)$ parameter $\mathrm{e}^{\mathrm{i}\omega}$, which labels the domain $\mathcal{D}_\omega$ of the Hamiltonian H_ω. The set $\mathcal{D}_\omega$ contains all the vectors of the form $\phi_+ + \mathrm{e}^{\mathrm{i}\omega}\phi_-$ together with the element of the domain $\mathcal{D}_0$. The solutions $\phi_\pm$ are

$$\phi_\pm = K_\gamma\left(r\,\mathrm{e}^{\mp \mathrm{i}\pi/4}\right), \tag{6}$$

where K_γ is the modified Bessel function [36]. The behavior of $\phi_+ + \mathrm{e}^{\mathrm{i}\omega}\phi_-$ near singularity $r \to 0$ is [37]

$$\phi_+ + \mathrm{e}^{\mathrm{i}\omega}\phi_- \simeq \mathcal{A}_+\left(\frac{r}{2}\right)^\gamma + \mathcal{A}_-\left(\frac{2}{r}\right)^\gamma, \tag{7}$$

where $\mathcal{A}_\pm = -\frac{\pi \mathrm{i}}{\sin(\pi\gamma)}\frac{\cos(\frac{\omega}{2} \pm \frac{\pi\gamma}{4})}{\Gamma(1\pm\gamma)}$.

We can now solve the eigenvalue problem (4). For $\gamma^2 \geq 1$ there are no bound states. More precisely there are no normalizable solution of the Schrödinger equation with negative energy. However, for $\gamma \in (-1,1)$, there is exactly one bound state with energy $\mathcal{E}$, where

$$\mathcal{E} = -\left[\frac{\cos\frac{1}{4}\left(2\omega+\gamma\pi\right)}{\cos\frac{1}{4}\left(2\omega-\gamma\pi\right)}\right]^{\frac{1}{\gamma}} \tag{8}$$

and the corresponding eigenfunction is

$$R(r) = K_\gamma\left(\sqrt{|\mathcal{E}|}r\right). \tag{9}$$

The asymptotic behavior of the bound state solution is exponentially decaying $R(r) \sim \mathrm{e}^{-|\frac{2ME}{\hbar^2}|r}$. Note that the bound state solution is obtained in [17,18] for relativistic particle moving in cosmic string background. Here we make some observation regarding our bound state solution (8) and (9). Note that imposing time-reversal symmetry in (3) we get [26]

$$\hat{T}\Psi(\phi+2\pi) = \mathrm{e}^{-2\pi\lambda\mathrm{i}}\hat{T}\Psi(\phi). \tag{10}$$

The consistency of (3) and (10) demands that either $\lambda=0$ or $\lambda=1/2$. However, $\lambda=0$ is not interesting because, in order to keep $\gamma=\frac{m+\lambda}{\alpha}$ in the interval $\gamma\in(-1,1)$ for $\lambda=0$, we have $m=0$. This makes the system independent of the effect of string characterized by α. So the interesting case for our purpose are (1) $\lambda=1/2, m=0$ and $\alpha\in(1/2,1)$ and (2) $\lambda=1/2$, $m=-1$ and $\alpha\in(1/2,1)$. The energy eigenvalue for $m=0,\lambda=1/2$ has been plotted in Fig. 1. For $m=0,\lambda=-1/2$, the plot will be same, because the eigenvalue (9) is symmetric with respect to γ. From Fig. 1, it can be seen that there may exist bound state for α arbitrarily close to unity.

The existence of bound state is in contradiction with the scale invariance, since scale invariance implies that there is no length scale in the problem, whereas the existence of bound state provides a scale. This can be resolved by looking at how scaling is implemented in the quantum theory. The scaling operator is

$$\Lambda = \frac{rp+pr}{2} \tag{11}$$

where $p=-\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}r}$. It is easily seen that Λ is symmetric on the domain $\mathcal{D}_0$ of H_r, and that for $\gamma^2\geq 1$, Λ leaves invariant the domain of the Hamiltonian. For $\gamma^2\in(-1,1)$,

$$\Lambda\phi = -\mathrm{i}\left(\phi+2r\phi'\right), \tag{12}$$

where ϕ is any element, belonging to the domain $\mathcal{D}_\omega$. The small r behavior of the function $\Lambda\phi$ is of the form

$$\Lambda\phi \simeq -\frac{\mathrm{i}}{2}\left[(1+2\gamma)\mathcal{A}_+\left(\frac{r}{2}\right)^\gamma + (1-2\gamma)\mathcal{A}_-\left(\frac{2}{r}\right)^\gamma\right],$$

where the constants $\mathcal{A}_\pm$ are defined above.

So, $\Lambda\phi$ clearly does not leave the domain of the Hamiltonian invariant. Scale invariance is thus anomalously broken [49–51], and this breaking occur precisely when the Hamiltonian admits nontrivial self-adjoint extensions. This also explains the quantum mechanical emergence of a length scale, namely the bound state energy. We must remark here that there does exist self-adjoint extensions that preserve scale invariance. For example, for $\omega=(1\pm\frac{\gamma}{2})\pi$ there does not exist any bound state. From the point of view of the domain, the operator Λ leaves the domain invariant.

Third, we come to the question of anomalous momentum transfer scattering cross section. We discuss this anomalous scattering in analogy with molecular physics [12,38–46]. In molecular physics, electron moving in the dipole field of a molecule experiences inverse square potential and electrons are loosely captured by this inverse square potential. It has been observed that the experimental value of the momentum transfer scattering cross section for the electron is much larger than the theoretically calculated value. It is usually argued[1] that the observed discrepancy between the experimentally observed and theoretically calculated scattering cross section of electrons is due to the formation of loosely bound state in the inverse square potential. The lifetime of these loosely bound electrons are very small. They are again released from the molecule and contribute to the momentum transfer scattering cross section. In our case also the situation is exactly same. The particle experiences the same attractive inverse square potential apart from the usual centrifugal term, while moving in the background metric of string. In this letter, it has been shown that particle can form loosely bound state in string defect background, see Fig. 1 for the behavior of bound state energy with respect to the constant parameter α, characterizing the string. So in same line with molecular physics we conjecture that the formation of loosely bound state in string background may lead to the anomalous momentum transfer scattering for the

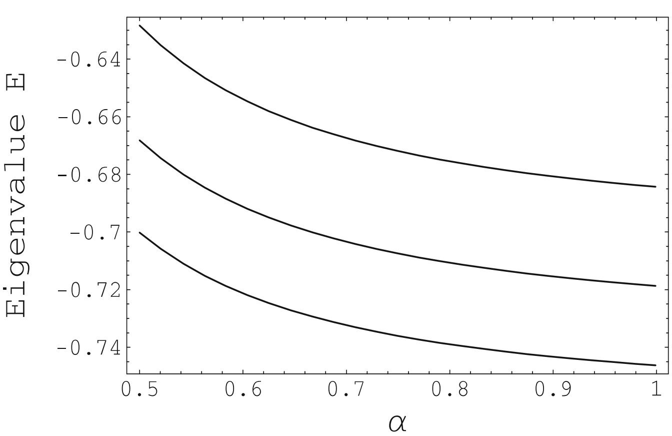

Fig. 1. A plot of bound state energy E (in $\frac{\hbar^2}{2M}$ unit) of particle as a function of α of the string defect for three different values of the self-adjoint extension parameter ω and with $m=0,\lambda=1/2$. From *top* to *bottom* $\omega=\frac{\pi}{7},\frac{\pi}{8},\frac{\pi}{9}$ respectively

[1] Anomalous scattering cross section for electron in the field of polar molecule due to temporary capture of the electron by the polar molecule was first pointed out by Turner. See [39] for detail discussion.

particles. However, the crucial difference between the molecular physics and our analysis is that in molecular physics the scaling symmetry explicitly breaks down in reality due to the finite size of the dipole but in case of string defect there seems to be no such explicit scaling symmetry breaking in non-relativistic theory. The behavior of particle in string background can be well formulated in condensed matter system [47] with defects and it could serve as the laboratory test for various properties of the string defect. In liquid crystal, for example, one can device such a laboratory test to observe the anomalous momentum transfer scattering due to the string defect in the crystal. One possibility is to observe the scattering of light from the core of string defect in transparent condensed matter system [48] and see the discrepancy with theoretical result, which does not take into account the effect of loosely bound state in scattering.

Before we conclude, we discuss about choosing the boundary condition or in other words choosing the value of self-adjoint extension parameter for string defect scenario. In [21] it is reported that for Schrödinger equation the physical boundary condition for cosmic string scenario is that the wavefunction is regular at origin. But for the relativistic case Dirac spinor is singular at origin, which is also supported by [17, 18, 20]. These results are based on a model where the flux is considered to be confined in a cylinder of radius R and then at the end taking the radius R to zero to find out the boundary condition. However there exists a different model described in [26, 27], where the the effect of string is considered within the metric of the spacetime. Since in our calculation we considered the effect of string within the metric like [26, 27], we consider the same boundary condition of [26, 27]. Thus unlike [21] according to our boundary condition, the wavefunction may be singular at the origin as long as it remains square integrable. Note that our boundary condition is more general and it includes the boundary condition of [21] as a special case.

In conclusion, we have shown the existence of scaling anomaly in string defect background. The consequence of this anomaly is the existence of bound state of the particle moving in string defect background. We have conjectured in analogy with molecular physics that there may have anomalous momentum transfer scattering due to the loosely bound state of the particle in string defect background. Although the conjecture is based on purely qualitative basis at this stage, it is however an interesting issue to study. In fact, in laboratory one can in principle devise experiment with condensed matter system to observe the anomaly in scattering due to the short lived bound state.

Acknowledgements. We thank P. Bhattacharjee for fruitful discussion with us and comments.

References

1. M.E. Peskin, D.V. Schroeder, An Introduction to Quantum Field Theory (Westview Books, New York, 1993)
2. S.B. Treiman, R. Jackiw, B. Zumino, E. Witten, Current Algebra and Anomalies (World Scientific, Singapore, 1985)
3. J.F. Donoghue, E. Golowich, B.R. Holstein, Dynamics of the Standard Model (Cambridge University Press, Cambridge, UK, 1992)
4. J.S. Bell, R. Jackiw, Nuovo Cim. A **60**, 47 (1969)
5. R. Jackiw, Lectures on Current Algebra and its Applications (Princeton University Press, Princeton, 1972)
6. S. Adler, Phys. Rev. **177**, 2426 (1969)
7. L.N. Epele, C.A.G. Canal, W.A. Ponce, Phys. Lett. B **411**, 159 (1997)
8. J. Polchinski, String Theory (Cambridge University Press, Cambridge, UK, 1998), Vol I and II
9. H.E. Camblong, L.N. Epele, H. Fanchiotti, C.A.G. Canal, Phys. Rev. Lett. **87**, 220 402 (2001)
10. H.E. Camblong, C.R. Ordonez, Phys. Rev. D **68**, 125 013 (2003)
11. T.R. Govindarajan, V. Suneeta, S. Vaidya, Nucl. Phys. B **583**, 291 (2000)
12. P.R. Giri, K.S. Gupta, S. Meljanac, A. Samsarov, Phys. Lett. A **372**, 2967 (2008)
13. M. Peskin, Phys. Rev. A **23**, 360 (1981)
14. V.B. Bezerra, J. Math. Phys. **38**, 2553 (1997)
15. C. Furtado, V.B. Bezerra, F. Moraes, Mod. Phys. Lett. A **15**, 253 (2000)
16. Y. Aharanov, D. Bohm, Phys. Rev. **119**, 485 (1959)
17. P. de Sousa Gerbert, Phys. Rev. D **40**, 1346 (1989)
18. B.S. Kay, U.M. Studer, Commun. Math. Phys. **139**, 103 (1991)
19. C.R. Hagen, Phys. Rev. Lett. **64**, 503 (1990)
20. W.B. Perkins, L. Perivolaropoulos, A.C. Davis, R.H. Brandenberger, A. Matheson, Nucl. Phys. B **353**, 237 (1991)
21. M.G. Alford, J. March-Russell, F. Wilczek, Nucl. Phys. B **328**, 140 (1989)
22. M. Reed, B. Simon, Fourier Analysis, Self-Adjointness (Academic, New York, 1975)
23. Dunford, J.T. Schwartz, Linear Operators, Spectral Theory, Self Adjoint Operators in Hilbert Space, Part 2 (Wiley-Interscience, New York, 1988) Wiley Clas edition
24. L. Feher, I. Tsutsui, T. Fulop, Nucl. Phys. B **715**, 713 (2005)
25. M. Alford, F. Wilczek, Phys. Rev. Lett. **62**, 1071 (1989)
26. C. Filgueiras, F. Moraes, Phys. Lett. A **361**, 13 (2007)
27. T.M. Helliwell, D.A. Konkowski, V. Arndt, Gen. Relat. Grav. **35**, 79 (2003)
28. P. de Sousa Gerbert, R. Jackiw, Commun. Math. Phys. **124**, 229 (1989)
29. S. Azevedo, F. Moraes, Phys. Lett. A **246**, 374 (1998)
30. S. Azevedo, J. Pereira, Phys. Lett. A **275**, 463 (2000)
31. S.R. Vieira, S. Azevedo, Phys. Lett. A **288**, 29 (2001)
32. S. Azevedo, Phys. Lett. A **293**, 283 (2002)
33. V. de Alfaro, S. Fubini, G. Furlan, Nuovo Cim. A **34**, 569 (1976)
34. B. Wybourne, Classical Groups for Physics (Wiley, New York, 1974)
35. K. Kowalski, K. Podlaski, J. Rembieliński, Phys. Rev. A **66**, 032 118 (2002)
36. M. Abromowitz, I.A. Stegun, Handbook of Mathematical Functions (Dover, New York, 1970)
37. B. Basu-Mallick, P.K. Ghosh, K.S. Gupta, Nucl. Phys. B **659**, 437 (2003)
38. E. Gerjuoy, Phys. Today **18**, 24 (1965)
39. J.E. Turner, Phys. Rev. **141**, 21 (1966)
40. G.S. Hurst, L.B. O'Kelly, J.A. Stokdale, Nature **195**, 66 (1962)

41. G.S. Hurst, J.A. Stokdale, L.B. O'Kelly, J. Chem. Phys. **38**, 2572 (1963)
42. S. Altshular, Phys. Rev. **107**, 114 (1957)
43. M.H. Mittleman, R.E. von Holdt, Phys. Rev. A **140**, 726 (1965)
44. K. Fox, J.E. Turner, Am. J. Phys. **34**, 606 (1966)
45. K. Fox, J.E. Turner, J. Chem. Phys. **45**, 1142 (1966)
46. J.M. Lévy-Leblond, Phys. Rev. **153**, 1 (1967)
47. M.O. Katanaev, I.V. Volovich, Ann. Phys. **216**, 1 (1992)
48. A.M. Srivastava, Phys. Rev. B **50**, 5829 (1994)
49. C. Duval, G.W. Gibbons, P. Horvathy, Phys. Rev. D **43**, 3907 (1991)
50. P.A. Horvathy, G. Morandi, E.C.G. Sudarshan, Nuovo Cim. D **11**, 201 (1989)
51. R. Jackiw, In: M.A.B. Beg: Memorial Volume, ed. by A. Ali, P. Hoodbhoy (World Scientific, Singapore, 1991)